Discrete RV X

pmf: $p_X(x) = P[X = x]$

$$P[a < X < b] = \sum_{\cancel{(x \in S_X)}\ a<x<b} p_X(x)$$

Continuous RV X

$P[X = x] = 0 \rightarrow$ pmf $\rightarrow$ useless

pdf (density): $f_X(x)$

$$P[a < X < b] = \int_a^b f_X(x)\,dx$$

$$-e^{-\infty} = -\frac{1}{e^{\infty}}$$

$$\int_0^{\infty} e^{-x}\,dx = \underbrace{-e^{-x}}\Big|_{x=0}^{x=\infty} = (0) - \left(-e^{-0}\right) = 1$$

Integration by parts

$$\int_0^{\infty} x e^{-x}\,dx = -e^{-x}(x+1)\Big|_{x=0}^{x=\infty}$$

$$= 0 - (-1)$$

$$= 1$$

$\frac{d}{dx}$			$\int$
x	$+$	e^{-x}	
1	$-$	$-e^{-x}$	
0		e^{-x}	

$$-xe^{-x} - e^{-x} = -e^{-x}(x+1)$$

$$\lim_{x\to\infty} -e^{-x}(x+1) = \lim_{x\to\infty} -\frac{(x+1)}{e^{x}} = -\frac{\infty}{\infty}$$

L'Hôpital $\Rightarrow \lim_{x\to\infty} -\frac{1}{e^{x}} = -\frac{1}{\infty} = 0$

Discrete

$$\mathbb{E}X = \sum_x x\, p_X(x)$$

$$\mathbb{E}[X^2] = \sum_x x^2 p_X(x)$$

$$\mathbb{E}[X^n] = \sum_x x^n p_X(x)$$

Continuous

$$\mathbb{E}X = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

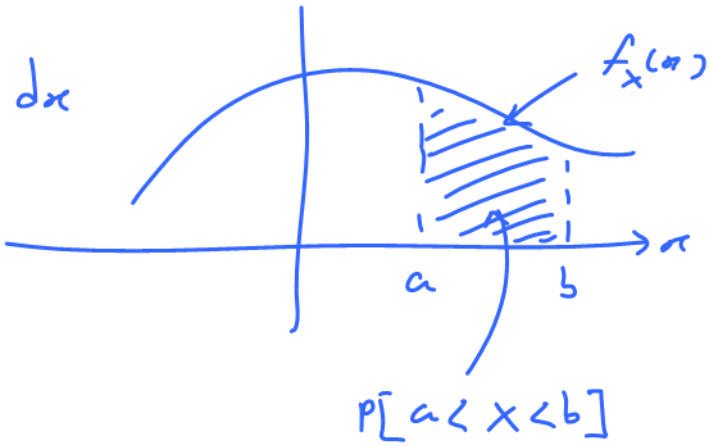
$$\mathbb{E}[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\,dx$$

$$\int_0^\infty x^2 e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^\infty = 0 - (-2)$$

$$= 2$$

x^2		e^{-x}	
	$+$ ↘		$-e^{-x}$
$2x$			
	$-$ ↘		e^{-x}
2			
	$+$ ↘		$-e^{-x}$
0	→		

$$P[a < X < b] = \int_a^b f_x(x)\,dx$$



$P[a < X < b]$

$$P[a < X < b] = \int_a^b f_x(x)\,dx$$

$$\approx f_x(a) \times (b-a)$$

$$\approx f_x(b) \times (b-a)$$

$$\approx f_x(x) \times (b-a) \qquad x \in (a,b)$$

$$f_x(x) \approx \frac{P[a < X < b]}{b-a}$$